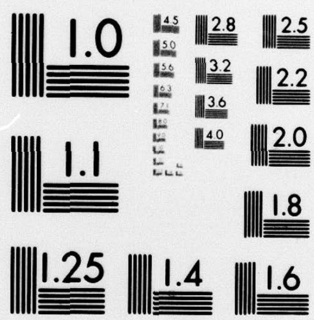
AD-A080 115   CALIFORNIA UNIV BERKELEY OPERATIONS RESEARCH CENTER   F/G 12/1
A NOTE ON FIRST PASSAGE TIMES IN BIRTH AND DEATH AND NONNEGATIV--ETC(U)
NOV 79   C DERMAN, S M ROSS, Z SCHECHNER   N00014-77-C-0299
UNCLASSIFIED   ORC-79-15   NL

1 OF 1
AD
A080115

END
DATE
FILMED
3-80
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

12

ORC 79-15
NOVEMBER 1979

A NOTE ON FIRST PASSAGE TIMES IN BIRTH AND DEATH AND NONNEGATIVE DIFFUSION PROCESSES

by

CYRUS DERMAN, SHELDON M. ROSS and ZVI SCHECHNER

OPERATIONS
RESEARCH
CENTER

UNIVERSITY OF CALIFORNIA · BERKELEY

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 14 ORC-79-15 ✓	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER: 9	
4. TITLE (and Subtitle): 6 A NOTE ON FIRST PASSAGE TIMES IN BIRTH AND DEATH AND NONNEGATIVE DIFFUSION PROCESSES.	5. TYPE OF REPORT & PERIOD COVERED: Research Report.
	6. PERFORMING ORG. REPORT NUMBER: 15
7. AUTHOR(s): 10 Cyrus Derman, Sheldon M. Ross [redacted] Zvi Schechner	8. CONTRACT OR GRANT NUMBER(s): N00014-77-C-0299 ✓AFOSR-77-3213B
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Operations Research Center ✓ University of California Berkeley, California 94720	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 16 2304/A5 17
11. CONTROLLING OFFICE NAME AND ADDRESS: United States Air Force Air Force Office of Scientific Research Bolling AFB, D.C. 20332	12. REPORT DATE: 11 Nov[redacted] 1979
	13. NUMBER OF PAGES: 8
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office): 12 9	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

61102F

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Also supported by the Office of Naval Research under Contract N00014-77-C-0299 with the University of California and N00014-75-C-0620 with Columbia University and the National Science Foundation under Grant MCS-7825-146 with Columbia University.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Birth and Death Process
Nonnegative Diffusion Process
First Passage Time
Increasing Failure Rate

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

(SEE ABSTRACT)

270 750

A NOTE ON FIRST PASSAGE TIMES IN BIRTH AND DEATH AND NONNEGATIVE DIFFUSION PROCESSES

by

Cyrus Derman
Department of Civil Engineering
and Engineering Mechanics
Columbia University
New York, New York

and

Sheldon M. Ross
Department of Industrial Engineering
and Operations Research
University of California, Berkeley

and

Zvi Schechner
Department of Industrial Engineering
and Operations Research
Columbia University
New York, New York

NOVEMBER 1979 ORC 79-15

This research has been partially supported by the Air Force Office of Scientific Research (AFSC), USAF, under Grant AFOSR-77-3213B and the Office of Naval Research under Contract N00014-77-C-0299 with the University of California and N00014-75-C-0620 with Columbia University and the National Science Foundation under Grant MCS-7825-146 with Columbia University. Reproduction in whole or in part is permitted for any purpose of the United States Government.

ABSTRACT

Consider a birth and death process starting in state 0 . ~~Keilson [1] has shown by~~ analytical arguments* that the time of first passage into state n has an increasing failure rate (IFR) distribution. We present a probabilistic proof for this. In addition, our proof shows that for a nonnegative diffusion process, the first passage time from state 0 to any state x is IFR.

* were shown earlier

1473A

A NOTE ON FIRST PASSAGE TIMES IN BIRTH AND DEATH AND NONNEGATIVE DIFFUSION PROCESSES

by

Cyrus Derman, Sheldon M. Ross and Zvi Schechner

0. INTRODUCTION

Consider a birth and death process starting in state 0. Keilson [1] has shown by analytical arguments that the time of first passage into state n has an increasing failure rate (IFR) distribution. We present a probabilistic proof for this. In addition, our proof shows that for a nonnegative diffusion process, the first passage time from state 0 to any state x is IFR.

1. THE RESULT

Let $\{X_j(t) \, , \, t \geq 0\}$, $j = 1,2$ be independent and identically distributed birth and death processes. Let

$$T_j = \min \{t : X_j(t) = n\} \; .$$

We start by showing that, given $T_j > t$, the conditional state at time t is stochastically increasing in the initial state.

Lemma:

For $i_2 > i_1$

$$P\{X_2(t) \geq k \mid X_2(0) = i_2 \, , \, T_2 > t\}$$

$$\geq P\{X_1(t) \geq k \mid X_1(0) = i_1 \, , \, T_1 > t\} \; .$$

Proof:

Assume throughout the proof that all probability statements are conditional on the events $X_2(0) = i_2$, $T_2 > t$, $X_1(0) = i_1$, $T_1 > t$; and consider sample paths for the two processes conditional on these events. Define

$$T = \inf \{s : X_2(s) = X_1(s)\}$$

and let it be ∞ if $X_2(s) > X_1(s)$ for all s ; and define the process $\{X_3(s) , s \geq 0\}$ by

$$X_3(s) = \begin{cases} X_2(s) & \text{for } s < T \\ X_1(s) & \text{for } s \geq T . \end{cases}$$

Now it is easy to see from the strong Markov property that $\{X_3(s) , s \geq 0\}$ has the same distribution as $\{X_2(s) , s \geq 0\}$. As $X_3(s) \geq X_1(s)$ for all s , the result follows.||

We now show that if $X_1(0) = 0$ and $T_1 > t$, then $X_1(t)$ increases stochastically in t .

Theorem 1:

$P\{X_1(t) \geq k \mid X_1(0) = 0 , T_1 > t\}$ is increasing in t .

Proof:

For $s < t$, we have by conditioning on $X_1(t - s)$ and using the Markov property that

$$P\{X_1(t) \geq k \mid X_1(0) = 0 \ , \ T_1 > t\}$$

$$= \sum_i P\{X_1(s) \geq k \mid X_1(0) = i \ , T_1 > s\} P\{X_1(t-s) = i \mid X_1(0) = 0 \ , T_1 > t\}$$

$$\geq P\{X_1(s) \geq k \mid X_1(0) = 0 \ , \ T_1 > s\}$$

where the last inequality follows from Lemma 1.||

Recalling that a continuous random variable X having distribution F and density f is said to be IFR if its failure rate function $\lambda(t)$ defined by

$$\lambda(t) = f(t)/(1 - F(t))$$

is increasing in t. If X is the life of an item, then $\lambda(t)$ can be interpreted as the instantaneous probability density that a t year old item will fail.

Corollary 1:

If $X_1(0) = 0$ then the first passage time to state n is IFR.

Proof:

It is easy to see that

$$\lambda(t) = \theta_{n-1} P\{X_1(t) = n - 1 \mid X_1(0) = 0 \ , \ T_1 > t\}$$

where θ_{n-1} is the birth rate at state $n - 1$. The result now follows from Theorem 1.||

Remark:

Our method of proof also shows that the first passage time from state 0 to any state x is IFR for any nonnegative diffusion process (that is, for any nonnegative Markov process on the real line whose sample path is continuous with probability 1). The key thing is that as in the case of birth and death processes, if we have 2 independent and identically distributed diffusion processes which start at time 0 in different states, then either the initially larger one remains larger or they meet at some point (since their sample paths are continuous) and if they meet their probabilistic structure is identical after they meet (by the strong Markov property). The above remains true even if we are conditioning on the event that the first passage time has not occurred by time t.

REFERENCE

[1] Keilson, J., "Markov Chain Models - Rarity and Exponentiality," ORC 74-32, Operations Research Center, University of California, Berkeley, California (October 1974).